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## RESEARCH SUMMARY

Major stand-level and forest-wide decisions facing forest managers interested in applying even-aged management are identified. Traditional, current, or potentially useful techniques for making these decisions are reviewed. Emphasis is placed upon how each method characterizes the stand or forest, the growth and yield information requirements of each method, the computational burden associated with each method, and the flexibility of each method and its adaptability to the form and complexity of the solution.

# **EVEN-AGED MANAGEMENT:**

## **Basic Managerial Questions and Available or Potential Techniques for Answering Them**

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# INTRODUCTION

Effective planning and conduct of a forest management program requires the forest manager to make a number of basic decisions; therefore a thorough understanding of the major questions prefacing these decisions is critical in forest management planning. Once the specific questions or decisions have been defined and understood, the forest manager chooses the particular analytical tool or tools to help in decisionmaking. Because a great deal of management science research has been devoted to developing alternative analytical tools, deciding which is best for a particular need can be difficult. The purpose of this study, therefore, is to examine the basic questions facing the manager interested in practicing even-aged forest management and to explain and evaluate the various analytical tools historically, currently, or potentially useful in answering the questions. Evaluation criteria will include: the way a particular method characterizes the stand or forest; the growth and yield information requirements of the method; the computational burden associated with the method; the flexibility of the method as to the form and complexity of the solution; and other pertinent considerations. A similar study has recently been reported for uneven-aged management (Hamm and Bare 1979).

## Management Questions

The basic timber management question is simply: What treatment should be applied to each stand each year to best meet the objectives established for the forest? The term "treatment" includes inaction.

This apparently simple question is actually very complex and difficult to answer. The manager is faced with a forest composed of perhaps hundreds of stands; a myriad of potential treatment types, intensities, and timings from which to choose for each stand; and also with an increasingly complex set of objectives and constraints for the forest. Even in the early days of forest management when forestry objectives were simpler and the set of possible treatments was relatively simple (that is, harvest cut or not), finding an answer to the basic question was difficult.

The basic question historically has been divided into a number of subordinate questions perhaps with the belief that answering subordinate questions would be easier. These subordinate questions can be separated into two categories: stand- (or aggregated stand-) level questions and forest-level questions.

For stands (or stand classes), the manager interested in practicing even-aged management needs to know the optimal:

1. Species mix
2. Planting density
3. Thinning plan
4. Fertilization plan and
5. Rotation length.

While it has not been done historically, 2, 3, and 5 from above could be combined into the question, "What is the optimal diameter or size distribution schedule?"

The term "stand classes" refers to the practice of aggregating stands of similar species mix, site quality, and age into classes to reduce the complexity of the problem. An "optimal" solution to a specified problem is defined to be that solution which best, or most favorably, meets the specified objectives and constraints of management. As defined, a plan includes both the schedule and intensity of treatment.

At the forest level, the forest manager interested in applying even-aged management is faced with determining, for the desired number of planning periods, the optimal:

1. Schedule of stand (or stand class) treatments and/or
2. Conversion strategy and conversion period length.

The two questions are closely related because a conversion strategy is also a schedule of stand treatments. The second question differs from the first if a targeted forest structure exists. Then the problem becomes how to best convert the present forest structure to the targeted structure while meeting the objectives and constraints of management. A regulated forest is an example of a targeted forest structure. Selection among alternatives for meeting one or several targeted forest structures constitutes the conversion strategy decision. The body of methods used are known as harvest scheduling techniques. Solution of these problems provides the allowable cut for each planning period.

The above questions can be regarded as basic questions as well as the set of problems dealt with in this paper. The techniques discussed in this paper can also be applied to make such subsidiary decisions as:

1. Selection of the most efficient logging techniques to achieve specific stand objectives (Sessions 1979)
2. Selection of the optimal harvesting schedule and of stand protection treatments in the face of such damaging agents as forest pests (Brodie and Rose 1975)
3. Selection of the optimal schedule over time for both stand treatment and roading access (Navon 1976).

These applications depend upon a thorough understanding of the basic stand treatment and harvest scheduling questions. They will be elaborated on further in this paper.

## TRADITIONAL SOLUTION METHODS

The two questions which have been studied the longest have been rotation length and the scheduling of stand class treatments to determine the allowable cut.

### Traditional Rotation Length Methods

As stated by Davis (1966), numerous criteria have been proposed to determine rotation length including: product size, culmination of mean annual increment, maximum forest rent, and maximum soil expectation value. The product size criterion assumes that the objective of management is to grow a certain size of tree (for example, pulp size trees or saw-log size trees). Rotation length is equivalent to the time it takes a stand of specified species mix and site quality to reach the desired size.

Culmination of mean annual increment (MAI) defines the age from time of regeneration, at which average annual growth, in volume, is maximum. Rotation lengths set to this age will maximize the physical productivity of the stand. The age at which culmination occurs will vary depending upon the species mix and site quality of the stand and upon the measure of volume. Culmination of MAI will occur sooner when expressed in total stem cubic foot volume than it will if expressed in board foot volume (Davis 1966).

Maximum forest rent is conceptually identical to culmination of MAI, the only difference being that the latter maximizes average annual volume growth while the former maximizes average annual net value growth. For a given stand age, annual net value growth is the gross value of the timber minus growing costs all divided by the stand age (Davis 1966). As an economic criterion for determining rotation length, this method has been strongly criticized for ignoring the opportunity cost incurred by locking up capital in a forest investment.

The final method, maximum soil (or land) expectation value, does include the opportunity cost of capital. The first step in computing rotation length using maximum soil expectation value is to determine, for a given rotation length and interest rate, the present net worth of all costs and returns, incurred and/or received during the rotation. Soil expectation value of the specific rotation length is then computed as the present value from receiving an infinite series of these present net worth "payments." This process is repeated for a number of rotation lengths and the rotation with the largest soil expectation value is chosen. The rotation of maximum soil expectation value will provide the owner with the highest return on investment in the property at the specified discount rate (Davis 1966).

## Evaluation of Traditional Rotation Length Methods

### CHARACTERIZATION OF THE STAND

Traditional methods require the stand to be characterized by those factors needed to obtain access to the particular yield tables used in the method's analysis. These factors usually include site quality, stand age, and species.

### GROWTH AND YIELD INFORMATION

Because all traditional methods use yield information, the assumptions incorporated in the yield information will subsequently be used in determining the rotation length. For example, if normal yield tables for unmanaged stands are used, it is assumed that the stand will be fully stocked throughout its life span and that no mortality capture by thinning will occur. On the other hand, if the yield tables incorporate specified thinning plans then it is assumed that the thinnings will occur on the specified schedules and with the specified intensity. While all methods do determine an optimal rotation length based on their specific criteria, it should not be assumed that the solution also represents the optimal treatment program for the stand. Rather, the solution is optimal for the given treatment program incorporated in the yield tables.

### COMPUTATIONAL BURDEN

All traditional methods can be done by hand. The product size criterion is the simplest method to apply, while the maximum soil expectation method requires the most information and is the most difficult to compute.

### FLEXIBILITY OF SOLUTION SPECIFICATIONS

Each traditional method has been designed to meet a specific objective. The objective of the product size criterion is to produce trees of a specified, fixed size. The objective of the maximum mean annual increment criterion is to produce the amount of wood volume possible, while maximum forest rent has the objective of the greatest average annual net value of the stand for a single rotation. Finally, the objective of the maximum soil expectation criterion is to produce the greatest net value of the stand for an infinite number of rotations.

## Traditional Methods for Scheduling Stand Treatments to Determine Allowable Cut

The other question that has historically received much attention is the scheduling of stand class treatments to determine allowable cut. Three methods have been used to find solutions to this problem--volume control, area control, and area-volume check.

Of the three methods, volume control requires the least amount of information to apply. No stand or stand class information is required and so allocation of the computed cut to actual stands must be accomplished after the fact. The goal of volume control is to provide a constant volume of cut over a specified period. Many formulas have been developed to compute the allowable cut in this fashion. Some of the best known include: the amortization formula, Von Mantel's formula, Grosenbaugh's formula, Hundeshagen's formula, the Austrian formula, Hanzlik's formula, Black Hills formula, and Kemp's formula (Davis 1966).

The objective of the area control method is to convert the forest to a regulated state within one rotation period by cutting equal areas in each planning period. For forests of homogenous type and productivity, the determination and allocation of the allowable cut is simple. After deciding upon a rotation length for the type, the number of acres cut each year is simply the total forest area divided by rotation length in years. The usual rule for allocating the area cut to the forest is: oldest stands are cut first. Therefore, by knowing the age and corresponding volume of each stand (or stand class), both the schedule of stand class treatments (assumed to be final harvest) and the volume removed in each planning period can be determined (Davis 1966).

If the forest is composed of a number of forest types and land productivity classes (which is most often the case), application of area control becomes more difficult. One traditional approach to this problem has been to express the different productivity and type classes in equivalent productivity units. The units are usually expressed as the fractional number of acres needed to produce a constant volume in regulated final harvest. Therefore, the method becomes productivity unit control instead of true area control (Davis 1966). Electronic computers have simplified the application of areas (or productivity unit) control to these more complex problems. Program AREA (Sassaman and Chappelle 1967) is an example of an area control computer program.

If a regulated forest structure exists, then both of the preceding control methods can provide a constant allowable cut, with equal areas (or productivity units) being harvested in each planning period. Regulated forests are, however, not usually encountered, which leads to problems. When applied to an unregulated forest, volume control results in an erratic number of acres being harvested while area control produces an erratic volume cut. The annual harvest calculated by one of the volume control formulas will be only a crude approximation of the true optimal harvest that can be sustained during conversion to a targeted forest. Because of these problems, a third methodology of scheduling stand class treatments to determine allowable cut has evolved, the area-volume check method.

The area-volume check method really covers a great number of techniques, all of which try to combine features from both the volume and the area control methods. Because of this diversity, it is difficult to generalize about the procedures. All methods do, however, require detailed information about the forest and its stands or stand classes, and assume that the conversion period equals the rotation length. Most methods also require a great deal of tedious calculation to determine an allowable cut figure. The last feature has made the introduction of the electronic computer to solve area-volume check problems a blessing to many foresters.

As an example of area-volume check, the following is a brief description of a method once widely used by Region 6 (Pacific Northwest) of the USDA Forest Service (Chappelle 1966). The first step in this method is to obtain an initial estimate of allowable cut through the use of volume control formulas such as Hanzlik's formula. The stand (class) data are arranged by cutting priority, which is usually "oldest stands first." The time required to harvest each stand class is then calculated using the initial allowable cut estimate. Given the initial age of the stand class and the time required to cut it, the expected yield from each stand class at final harvest is calculated by projection. After harvest of all initial stand classes in the forest, two checks are made. The first examines whether the actual time needed to completely harvest the forest equals the desired conversion period and the second examines whether the acreage harvested each year is acceptable. If either of these checks are unacceptable, then the allowable cut is changed in a systematic fashion and the process is repeated. Chappelle (1966) incorporated these steps in the computer program ARVOL to actually perform the operations.

## Evaluation of Traditional Stand Treatment Scheduling Methods

### CHARACTERIZATION OF THE FOREST

Volume control through formula does not need to characterize the forest because it does not provide a schedule of stand treatments. The other two traditional methods characterize the forest by lumping stands into rather broad classes that are usually defined by age and perhaps site quality.

### GROWTH AND YIELD INFORMATION

Treatment plans and their yields are fixed for each application of the method. Except for repeated solutions, no choice of how a stand class can be treated is provided; for example, runs with and without thinning might be compared. In addition, thinning cuts can only be expressed as increases in final harvests or as priority harvests (that is, thinning harvests must be taken before regeneration harvests).

### COMPUTATIONAL BURDEN

All of the traditional methods can be computed by hand. Volume control through formula is the easiest to compute, while the area-volume check method is the most difficult and tedious technique to apply.

### FLEXIBILITY OF THE SOLUTION SPECIFICATION

None of the traditional methods can handle economic criteria for stand treatment scheduling.

### OTHER CONSIDERATIONS

Both area control and area-volume check assume that the conversion period equals the rotation length and that a targeted forest structure exists. In addition, the harvest priority of their stand classes must be set before applying the two methods. Such rules as cut oldest stands first or cut highest volume stands first determine the harvest priority.



# OPERATIONS RESEARCH METHODS

The introduction of both the electronic computer and operations research techniques greatly expanded the manager's ability to answer questions pertaining to even-aged stands and forests. Some of the first applications of the computer were to mechanize the computations of numerical values previously, and tediously, done by hand, such as the aforementioned allowable cut methodologies. Two broad areas of operations research have seen application to forestry problems, simulation and optimization.

The distinction between simulation techniques and optimization techniques in even-aged stand and forest-level analysis is not distinct because of confusion in usage. Confusion arises from a body of intermediate methods lumped together under the term "binary search techniques." We will distinguish between pure simulation, pure optimization, and binary search techniques. For the purpose of categorization, we will, however, follow the usage of Johnson and Tedder<sup>1</sup> and lump the binary search techniques under simulation.

A pure simulation approach requires that the analyst specify the entire treatment plan for each stand. The simulation tool used provides a detailed report of the impacts of these decisions. Sensitivity to changes in these decisions is provided by repeated simulation analysis under alternate treatment plans. The alternative selected would be the best of all alternatives tried, judged against some implied objectives and set of constraints. Superior alternatives could easily exist, but may be missed.

Under a pure optimization approach, the analyst supplies only a specific objective and a set of alternatives and constraints. The optimization technique then proceeds to find the best level of the objective criterion that can be met under the specification. No superior solution to this particular specification will exist. Of course, the analyst can check the sensitivity of his plan to alternate specifications of the problem through additional output provided or by resolving the problem under alternate specifications. Pure optimization applications to even-aged management have used mathematical programming techniques including: dynamic, linear, nonlinear, goal, and integer programming. These applications will be discussed.

Binary search techniques codify the repeated trials implicit in the pure simulation approach. Advantage can be taken of knowledge about the structure of forest management problems to efficiently solve some but not all potential problem specifications. The binary search techniques will give answers identical to the pure optimization techniques under certain objective criteria and constraint ranges, but different and lower valued solutions under others. Currently, binary search techniques provide greater flexibility in handling complex sets of regional inventory data involving multiple ownerships and forest types with different allowable cut assumptions.

## SIMULATION

### Simulation Methods for Answering Stand-Level Questions

A good deal of work has been done in applying pure simulation to answering stand-level questions. As examples, the work of Hamilton and Christie (1974) in Great Britain, of Myers (1969, 1973) in the Rocky Mountain area, and of Hoyer (1975) in Washington are of interest. All three methods use a stand development model in a manner which allows the user to modify the thinning plan and the rotation length in order to analyze the possible consequences of the specified management program. Each simulation run produces

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<sup>1</sup>Johnson, K. Norman, and Philip L. Tedder. Linear programming vs. binary search in allowable cut calculation. For. Sci. [in preparation].

information concerning stand development, yields from thinning and harvest cuts, and present net worth based on discounted costs and revenue figures. This information is reviewed by the user (the forest manager) to see if the particular solution looks "best" when compared to other selected management programs.

In addition to examining thinning plans and rotation lengths, Hamilton and Christie (1974) provide the user with an opportunity to examine initial planting densities, while Hoyer (1975) provides the means of examining the effects of fertilization at one intensity.

## **Evaluation of Simulation Methods for Answering Stand-Level Questions**

### **CHARACTERIZATION OF THE STAND**

The particular data items used to characterize the stand depend upon the type of stand development model used by the simulation method. These data items can range from site index, stand age, and stand basal area and/or total number of trees to site index, stand age, and the size and competitive position of individual trees within the stand.

### **GROWTH AND YIELD INFORMATION**

Stand simulation methods can be applied to any kind of existing stand development model. With the use of more current stand development models, it is often possible to predict yields from a wide array of treatment plans.

### **COMPUTATIONAL BURDEN**

Most stand simulators require a computer to operate.

### **FLEXIBILITY OF THE SOLUTION SPECIFICATION**

While the stand simulation methods can be used to assess either physical or economic objectives, the methods cannot assure that the examined management program represents the "global" optimal solution to any specified set of complex objectives, constraints, and cost-revenue relationships. To determine a global or near global optimal solution would require a great number of simulations and careful analysis and can be accomplished more efficiently by optimization for many stand development models.

## **Simulation Methods for Answering Forest-wide Questions**

Simulation has also been used extensively in answering forest-level questions, even before the introduction of the electronic computer. The area-volume check method is an example of a precomputer binary search simulation method. The computer has, of course, increased the manager's capability to utilize simulation. It has shortened the time and tedium associated with obtaining a solution. Examples of where binary search simulation has been employed for managerial questions are the programs: SORAC (Chappell and others 1972), and TREES (Tedder and others 1972). The application of pure simulation is program TEVAP2 (Myers 1974).

Program SORAC can compute an allowable cut and area-volume check. The program allows the manager to select a period in the planning horizon. If area control is selected, then the solution is identical to that obtained with area control parameters that change between periods will differ from AREA. When the area-volume check method is selected, the results from SORAC also differ from AREA.

Program ARVOL computes an allowable cut that can be sustained for the conversion or rotation period. Using this method, however, can cause the allowable cut to change drastically after the conversion period or first rotation. Also, a manager rarely computes an allowable cut that will be followed for an entire conversion or rotation period. Instead, he will apply the cut during the current planning period and recompute a new allowable cut figure at the start of the next planning period. Using this continual updating feature, drastic changes in allowable cut from one planning period to the next can be avoided. SORAC basically tries to simulate this actual planning and updating process. While the allowable cut for the current planning period is the same as that provided by ARVOL, the advantage of SORAC over ARVOL is that SORAC provides the manager with a more realistic picture of the long-term consequences of his actions.

Like ARVOL and AREA, SORAC characterizes the forest by stand classes that are usually based on stand age. Finally, either extensively or intensively managed yields can be specified for each stand class, with intensive yields implying a predetermined treatment schedule.

In program SIMAC, the forest is also characterized by stand age classes, with a user-specified average site index for the whole forest. SIMAC, however, allows much more flexibility in specifying a treatment scheme for testing than SORAC. SIMAC also differs through its allowable cut procedures. First, no rotation or conversion period length is specified. Instead, the length of the planning horizon is specified (up to 40 decades) along with the youngest age in which a stand class can be regeneration harvested. To meet the allowable cut, the simulation can harvest a stand class at any age down to the minimum age (that is, final harvest age is not predetermined).

The second difference is that an age-related cutting priority can be specified instead of assumed. A third difference is that a fixed allowable cut can be specified for any or all of the first 10 decades. Finally, an upper and lower bound on allowable cut can be specified and the SIMAC program will compute, if possible, the maximum allowable cut per decade that can be removed over the planning horizon and still be within these bounds. The allowable cut technique used by SIMAC is a systematic trial and error process like that used by ARVOL. In fact, the allowable cuts would be identical in both if the bounds did not interfere with the process (and if the user did not require specific allowable cuts in the first 10 decades).

The TREES model (Tedder and others 1980) is a package of binary search techniques embodying the alternatives of area control, volume control, SIMAC, SORAC, or ECHO type approaches (ECHO is discussed under optimal forest-wide decisionmaking methods). A high degree of flexibility in specification can be maintained for:

1. Harvest priority;
2. Variable silvicultural intensity over time and portions of the inventory;
3. Alternate independent or interdependent methods and criteria for determining the harvest for subsets of the inventory such as public or private;
4. Resolution of results by timbershed, ownership, species, site, and prescribed silvicultural treatment.

The higher degree flexibility than that available from the pure optimization packages presently available has resulted in these techniques being chosen for detailed regional projections (Beuter and others 1976).

The final example of an allowable cut simulator is program TEVAP2. Besides being a pure rather than binary search method, this program has a number of features which distinguish it from SORAC, SIMAC, and TREES. First, TEVAP2 deals with stands rather than stand classes. The stands must be even-aged with one or two stories, but they can be uniquely characterized by their type; site index; and the average diameter, height, number of trees, age, and dwarf mistletoe rating of the overstory and understory. For this reason, average or "normal" stands are not assumed and treatments can be better tailored to the actual stand. It should be noted that Myer's (1968, 1973) stand simulators can easily be used to derive the treatment schedule to be used in this forest-level analysis.

The second distinguishing feature of TEVAP2 is its planning horizon. In TEVAP2, the planning horizon is equal to the planning period and, for many users, the planning period is one decade. TEVAP2, therefore, is a short-range planning tool when compared to SORAC, SIMAC, and TREES.

Allowable cut determination methodology is the final distinguishing feature of TEVAP2. Four allowable cut levels are computed for the planning period. The first allowable cut figure is calculated under the assumption that all specified treatments are applied to optimally stocked stands existing in a forest with a perfectly balanced age structure. The second allowable cut is calculated under the assumption that all specified treatments are applied to the existing stands and forest structure. The third cut is computed by applying area regulation to the existing forest structure. The final allowable cut value is computed by volume control using a modified version of the Austrian formula.

The allowable cut figures are not applied to the existing forest structure for the conversion or rotation length to determine if they can be sustained. Rather, these cuts serve as guides and the forest manager can redefine treatment schedules and intensities if he is not pleased with the results.

## Evaluation of Simulation Methods for Answering Forest-wide Questions

### CHARACTERIZATION OF THE FOREST

Some forest-wide simulation methods (for example, SORAC and SIMAC) characterize the forest by combining stands into classes using stand age, site quality, and/or species. Other forest-wide simulation methods (for example, TEVAP2 and TREES) characterize and maintain each stand of the forest using detailed stand information.

### GROWTH AND YIELD INFORMATION

Some forest-wide simulation models, such as SORAC, SIMAC, and TREES, use tabular yield information which can be derived from any yield source. Other forest-wide simulation models, such as TEVAP2, have stand development models built into stand development models usually have been of the whole stand type. Unlike traditional forest-wide decision methods, all of the aforementioned forest-wide simulation methods, except SORAC, allow thinning yields to be credited to total harvest when they occur instead of at the final regeneration harvest cut.

### COMPUTATIONAL BURDEN

All of the forest-wide simulation methods used as examples require a computer to operate. When compared to forest-wide optimization techniques, simulation methods are often faster to run on the computer and therefore cheaper (Johnson and Tedder<sup>1</sup>).

## FLEXIBILITY OF THE SOLUTION SPECIFICATION

Simulation methods SORAC, SIMAC, and TEVAP2 are used exclusively for seeking the objective of physical wood production. Program TREES can be used to meet either physical or economic objectives. As with the stand-level simulators, the treatment schedules and intensities are fixed for the run while cutting priority may be fixed by the simulation method. To determine a truly global, optimal solution would require repeated trials with different schedules, intensities, and priorities. If the potential number of treatment types is large, including such types as planting intensity, thinning, fertilization, harvesting method, and species mix, finding a global solution may be extremely difficult.

Another limiting feature of the forest-wide simulation methods is the difficulty of introducing constraints into the analysis that might be applied to the harvest flow. Simulation models, however, can more readily identify feasible solutions than their optimization counterparts (Johnson and Tedder<sup>1</sup>).

## OPTIMIZATION METHODS AT THE STAND LEVEL

Optimization methods for solving stand-level questions have long been used in forestry. Early methods of determining rotation length, such as maximum MAI and soil expectation were all optimization methods. These early methods were distinguished by the numerous assumptions made in order to solve them and by the amount of tedious calculations required. First applications of the electronic computer were aimed at solving the latter problem. More recently, mathematical programming techniques have made it easier to solve more complex problems and, therefore, reduce the need for numerous assumptions. A good example of this evolutionary process has been the development of optimization tools for jointly answering thinning plan and rotation length questions.

### Optimal Thinning Plan and Rotation Length

Chappelle and Nelson (1964) presented an early solution to the joint optimization of thinning plan and rotation length in loblolly pine. They used volume growth equations and marginal analysis to determine optimal volume stocking levels which would maximize net profit per annum. Then given an initial stocking level, the optimal stocking level, and the volume growth model, they determined the volume to be removed by thinning in each cutting period for a fixed rotation length. Using this information and cost and revenue data, the optimal rotation length was determined using the classical soil expectation value method.

The solution computed by Chappelle and Nelson (1964) could be found by hand, but they computerized the process for ease of operation. While their method was advanced for its time, a number of assumptions or simplifications were made to reduce the computational load. First, the growth models were very simplistic. Second, the stumpage price was assumed to be either constant over size of material or a linear function of stand age. Finally, costs that varied with stocking were not treated directly in their method (that is, stumpage was assumed to be net instead of gross).

In a later development, Amidon and Akin (1968) demonstrated how dynamic programming could be used to obtain the same solutions as Chappelle and Nelson (1964). Dynamic programming is a discrete mathematical programming tool that can describe a problem through a network of points. The number of "state" descriptors needed to define each point also defines the dimensions of the network. For example, Amidon and Akin defined each point by its volume stocking, with 1,000 board foot intervals between points, and by its stand age, with 5-year intervals between points. Their network, therefore, had two state descriptors and was two dimensional.

The objective of dynamic programming is to find, within defined limits, the best or optimal path through the network. In the Amidon and Akin (1968) problem, therefore, the objective was to find the optimal stocking level at each age class. The limits of the network are defined by physical considerations such as maximum growth of the stand and the impossibility of negative volumes. Solution to the problem of finding an optimal path can be accomplished using either the forward or the backward recursive method.<sup>2</sup>

Because the forward recursive method is easier to understand, it will be explained first using the Amidon and Akin problem as an example. An initial stand age and stocking level are specified which defines the starting point in the network. A growth function is then used to advance the stand to one of the points in the next time period. If nothing were done the stand would remain at the stocking level of the point to which it had grown. Thinning, however, can reduce stocking which shifts the stand to other "lower" points in that time period. For the second time period, there is only one path the stand could have followed to get to each point because of the single starting point in the first time period. Because the second period has defined a number of points that the stand can grow from, there are a number of different paths in which a stand could arrive at a stocking point in the third time period. What is wanted, though, is the optimal path to that point. Therefore, all possible paths from one age to the next are examined and only information for the optimal path is stored for each possible stocking level at time period three. This process is continued for all stand age classes in the network.

The values that must be stored at every point in the network, excluding the starting point, are the value of the objective function for the best path to that point and the coordinates of the previous point on the optimal path. With the forward recursion method, therefore, it is possible to examine all possible points at a given age and, using the objective function value, determine the optimal stocking level for that age. Given that point, the optimal path (and therefore stocking level for each age) can be determined from the stored coordinates.

Inherent in this method of determining the optimal path is the "principle of optimality." Translated into the problem of determining the optimal thinning plan, this principle states that, once the optimal thinning plan has been determined to a specified stand age and structure, the optimal plan for the next older stand age depends only on the older stand's age-structure combinations not yet analyzed. Thus, the various possible stand structures for a given age need to be analyzed only once, which greatly reduces the number of calculations necessary.

The backward recursion method ultimately provides the same answers as the forward recursion method, but the values stored at each point are different. The backward method starts at all possible ending points and determines the best path to the starting point. Stored at each point, therefore, is the optimal path to value of the objective function.

The advantage of the backward recursion method from the optimal path due to insects, fire, or already computed will show which new path is of age. The disadvantage (and therefore the advantage) is that a new run must be made for each potential

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<sup>2</sup>It is our belief that dynamic programming is a difficult tool to understand for many in our proportionally more attention to it.

Amidon and Akin (1968) used the backward recursive method. They also greatly restricted the problem by allowing the stand to increase or decrease only 1,000 board feet in one growth period (this is, between adjacent age periods). This meant that a given point could only be reached from two adjacent points. While this reduced the complexity of the problem and made solution easier, it also greatly restricted possible thinning plans.

Because the backward recursive method solves for only one rotation length in a given run, Amidon and Akin (1968) had to make a number of runs to evaluate the effect of changing rotation length. Like Chappelle and Nelson (1964), they used the maximum soil expectation criterion for determining the optimal rotation length.

As pointed out by Amidon and Akin (1968), the advantage of dynamic programming over the marginal analysis approach of Chappelle and Nelson (1964) is its computational flexibility. Once the problem has been structured for dynamic programming, it is easy to change pricing and cost assumptions. Also, the results of one run can be used to evaluate other "suboptimal" solutions that might become viable if stand conditions change. A final computational advantage is that stand development models and dynamic functions need not be as simple as those used by Chappelle and Nelson (1964) and Amidon and Akin (1968).

After examining the approaches of Chappelle and Nelson (1964) and Amidon and Akin (1968), Schreuder (1971) made two critical comments on their methodology. First, he contended the four had ignored cost of land when determining optimal stocking levels. This omission, he said, can affect the determination of economic stocking levels. Second, he felt that their method would not find the jointly optimal thinning plan and rotation length because their method "...does not consider the possible interdependencies of the period." Therefore, their method would ignore thinnings with negative value for the period but which, over the rotation period, could more than pay for themselves. Their method would also ignore timber selling costs, which vary with volume removed. Both of these factors could lead to suboptimal paths.

Schreuder (1971) proposed that thinning schedule, intensity, and rotation length could be easily optimized jointly if the final harvest cut were defined as an extreme thinning. He formulated the joint optimization problem in a continuous, control-theoretic form in which revenue, costs, rate of timber cut, stand yield, and growth were all functions of time. This formulation would determine the optimal thinning and quantity of volume removed in continuous, instead of discrete, values of time and volume. Once he had the formulation, however, he discovered that an explicit solution could not be found except for trivial problems. Schreuder (1971) then turned to the numerical solution method of dynamic programming to find a discrete solution to his formulation. Although he formulated the solution in a backward recursion, dynamic programming framework, he did not show actual examples of the process.

Practical application of dynamic programming to solve the joint optimization problem has recently been reported by Brodie and others (1978) and by Brodie and Kao (1979). In addition, Brodie and others (1978) demonstrated that, contrary to Schreuder's (1971) findings, the approaches of Chappelle and Nelson (1964) and Amidon and Akin (1968) do indeed simultaneously solve for both optimal stocking level and rotation length.

The purpose of the Brodie and others (1978) study was to assess the "impacts of regeneration cost, initial stocking level, site, quality premiums, and variable logging costs on thinning and rotation...." Three features distinguish this dynamic programming formulation from that of Amidon and Akin (1968). First, the stand model used by Brodie and others (1978) incorporates a mortality estimator. More realistic estimates of the effects of thinning, through mortality capture, therefore, can be made and evaluated.



Second, the dynamic network of Brodie and others (1978) allows much greater accuracy in defining thinning intensity than did Amidon and Akin's (1968) network. From any specified node, the latter's network allowed the stocking level to either increase or decrease 1,000 board feet. Brodie and others (1978) allow stocking to decrease by 100 cubic foot increments to zero and to increase by 100 cubic foot increments to maximum potential stockings for the age class.

Finally, Brodie and others (1978) chose to use the forward recursion method instead of the backward as advocated by Amidon and Akin (1968) and Schreuder (1971). For thinning analysis, they considered the forward method a more flexible tool than the backward method.

One shortcoming that Brodie and others (1978) identified with their approach was that the stand model did not provide diameter growth acceleration usually expected from thinning. If quality premiums are important, this shortcoming can lead to suboptimal solutions.

The most recent work in the application of dynamic programming to stand problems, by Brodie and Kao (1979), eliminates this shortcoming. They accomplished this by using a much more complex stand model that incorporates growth acceleration in the quadratic mean stand diameter. After a careful examination of the stand model, Brodie and Kao (1979) found that the model could be initialized for a specified site index if the three values of stand age, basal area, and number of trees were known. These variables, therefore, formed the three state descriptors for the dynamic programming solution. The sizes of the intervals between nodes are 10 years, 4 square feet of basal area, and 15 trees, respectively.

Analysis results using the three-descriptor framework are the optimal number of trees and basal area to maintain in each time interval. To obtain these values, Brodie and Kao (1979) used the forward recursion solution method. Because quadratic mean stand diameter can be computed from number of trees and stand basal area, stumpage prices and logging costs could be more realistically introduced into the analysis as functions of quadratic mean stand diameter.

## Other Possible Methods of Determining Optimal Thinning Plan and Rotation Length

The dynamic programming approaches reported to date have all ignored or assumed a diameter distribution structure. Diameter distribution data is important, however, in the planning of milling facilities and as an aid in applying specified treatments to field conditions. Therefore, a procedure that can determine the optimal number of trees for each time period by diameter classes may have greater usefulness than present procedures.

Conceptually, dynamic programming can also be used to determine optimal class distributions over time. The state descriptors would be number of trees ( $T_c$ ) in one of the  $D$ th diameter classes at one of the  $C$ th number of dimensions for the network space is  $D+C$  network is  $A(T_c)^D$ . From these formulas, it is easy to see that it fast becomes impossible in practical applications. To determine, within 10 trees per diameter class, the optimal number of trees to maintain in each 2-inch diameter class at the start of each time period, that the maximum number of trees per diameter class was 120 years, and the maximum diameter class size was 40 inches, the number of classes would be 26 (0, 1-10, 11-20, ..., 241-250), the number of diameter classes 20 (2, 4, ..., 120), and the number of dimensions for the network space would be  $C=26$ . The number of nodes in the network would be  $12 \times 26^{20} = 3.12 \times 10^{22}$ .



Discussion in this section has been confined to deterministic analysis of stand-level questions. Stochastic stand-level decision analysis using dynamic programming is presented in several papers (Hool 1966, Lembersky and Johnson 1975; Lembersky 1976).

Another potentially promising approach has been described by Adams and Ek (1974). Part of their work dealt with the problem of converting an existing uneven-aged diameter distribution to a targeted structure over a number of time periods in which cutting could occur. This problem is analagous to the even-aged situation of starting with a specified stand, growing, and thinning it over a number of time periods until a targeted diameter distribution of zero trees in each diameter class is reached in the last time period. Adams and Ek (1974) used a nonlinear programming approach which necessitated a relatively simple stand model that explicitly predicted the net change in number of trees in a diameter class. Their method was also limited as to the number of time periods over which the method could be applied.

It is obvious that methods for determining what diameter distributions are optimal to carry over time are in their infancy. Alternatives to dynamic or nonlinear programming, such as application of optimal control theory or decomposition theory, have been suggested (Adams 1974 and Adams and Ek 1976), but the development and testing of practical applications is limited.

## Optimal Fertilization Plan and Species Mix

Fertilization and species mix are two additional factors which can interact and affect optimal thinning plans and rotation lengths. Consequently, they should all be jointly optimized to reflect their interdependencies. It has already been shown how an optimal thinning plan, and rotation length can be jointly determined for a stand. The problem now is the incorporation of fertilization and species mix into the process.

The optimization of timing and intensity for fertilization has not been addressed in the literature. This is explained by the scarcity of stand models with built-in fertilization response functions. Once appropriate stand models have been developed, the joint optimization of thinning schedule and intensity, rotation length, fertilization schedule, and intensity can be done using dynamic programming. Incorporation of fertilization intensity results only in a multiplicative increase in computations and not an exponentiation of network size,<sup>3</sup> provided that entry-time intervals are longer than the effect of fertilization.

Species mix is another area where little work has been reported. As with fertilization, one reason for this is the dearth of stand simulators that can handle multiple species. Given a multiple species stand model, optimization could be done by use of dynamic programming. An additional state descriptor would be required for each species in the problem.

## Evaluation of Optimization Methods for Answering Stand-Level Questions

### CHARACTERIZATION OF THE STAND

As with the stand-level simulation methods, all stand-level optimization methods characterize the stand by those features used to initialize the particular stand development model type used by the optimization methods.

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<sup>3</sup>Kao, Chiang. 1980. A study of optimal timing and intensity of silvicultural practices--commercial and precommercial thinning, fertilization, and regeneration effort. Ph.D. diss. Dep. of For. Manage., Oreg. State Univ., Corvallis. 219 p.

## GROWTH AND YIELD INFORMATION

Stand-level decision using dynamics programming can be applied to any type of stand development model, if it is acceptable to the manager to prespecify thinning rules (such as, cut smallest or largest trees first) for priority of removals (Martin 1978). If this condition is not acceptable, then application of dynamic programming is currently limited to the whole-stand/diameter-free type of stand development model.

Analyses using nonlinear programming, and perhaps control theoretic techniques, are potentially applicable to the whole-stand/diameter-class type of stand development model.

## COMPUTATIONAL BURDEN

In application of dynamic programming, size of the problem can cause difficulties and pose limitations. For example, if a stand simulator existed that was initialized by number of trees and basal area for any of a number of species and if A is the number of age classes, T is the number of tree classes, B is the number of basal area classes, and S is the number of species, then the dimensional size of the dynamics programming network space would be  $2S+1$  and the number of nodes would be  $A \cdot (B \cdot T)^S$ . Thus, problems of this type become large very fast and, if a diameter class distribution type of simulator were used, the size of the problem is increased many fold (that is, using these and the previously defined symbols, size of the network space would be  $SD+1$  and number of nodes would be  $A \cdot T_c^{S \cdot D}$ ).

Another limitation of optimization methods is the size of their computer programs. This problem is particularly aggravated if the optimization method is interfaced with some of the complex stand development model types.

## FLEXIBILITY OF THE SOLUTION SPECIFICATION

Both the dynamic programming and the nonlinear programming methods can determine the optimal treatment plan to follow to meet both physical and economic management objectives.

## OTHER CONSIDERATIONS

As a precaution, the "principle of optimality" that is inherent in dynamic programming is a restriction that must be dealt with carefully. It has been previously shown how a model that characterizes stand development by site index, age, basal area, and number of trees per acre can be used to determine optimal thinning plans by the dynamic programming technique. Because the time variant variables (age, basal area, and number of trees) were all state descriptors, the "principle of optimality" can be met. If additional variables are used in the model, then one of three actions can be taken to still meet the principle. First, the additional variables can be added to the analyses as additional state descriptors. This is the most obvious and theoretically best solution, but, as demonstrated, it comes at the potentially high cost of requirement of larger computer capacity.

The second method of handling additional variables is to make assume the relationship between the additional variable(s) and the state descriptors was the method used by Martin (1978). Finally, for some variables, it is possible to avoid the problem by modifying the existing state descriptors. Suppose that additional variables (such as time since last cutting or time since last thinning) have been used to explain variation not explained by the state descriptors (residual basal area and/or number of trees). If the effect of these variables is zero after a period of time, then these variables can be handled, without additional state descriptors, by defining the planning period length to be as long as or longer than the period it takes for the effect to disappear.

## OPTIMIZATION METHODS AT THE FOREST LEVEL

Over the past two decades, a considerable amount of effort has been directed at developing techniques for answering forest-wide management questions. An excellent evaluation, analysis, and summary of most of these techniques has been provided by Johnson and Scheurman (1977). They defined two basic mathematical programming formulations, which they called Model I and Model II, and two pricing options. The two basic formulations differ by their treatment of the stand classes. For the sake of simplicity, Johnson and Scheurman (1977) assume that the forest is composed of one species and one site quality and that the resulting forest structure can be adequately portrayed by age classes.

With this forest structure, Model I defines a management unit by its initial age class and these units are maintained throughout the analyses. Therefore, only one set of activities consisting of a number of alternative treatment plans must be defined for each management unit. A treatment plan is defined by the timing of harvest and thinning cuts and the resultant returns expected across the entire planning horizon. The optimal solution is characterized by the number of acres in a management unit assigned to each treatment plan in order to meet management objectives and constraints.

Model II also defines starting management units by their initial age classes. New management units, however, are defined if harvesting of the initial age classes occurs (that is, if harvesting occurs in several different age classes during a period, then all acres harvested are combined into a new management unit). As a result, management units are not fixed for the entire analysis.

For Model II, a number of activity sets, or treatment plans, must be defined. A first set of activities defines for the initial management units the returns from all possible timings of the first harvest cut or of the return at the end of the planning horizon if the units are never harvested. A second set of activities defines for management units that have been cut and defined due to first harvesting all possible timings of the second harvest cut or of the returns at the end of the planning horizon if the units are never reharvested. The required number of sets of activities of this type is equal to the maximum number of harvest cuts that can be made in the planning horizon to a single stand class.

While only one set of activities is needed for the Model I formulation, Johnson and Scheurman (1977) have shown that the number of activities in this set could greatly exceed the total number of activities in all of the sets of the Model II formulation. On the other hand, Model II requires more area constraints than the Model I formulation. Area constraints guarantee that the area harvested is no greater than the area within an age class in any period. Nonnegativity constraints guarantee that harvest volumes and areas from a class are not negative. Other constraints, such as harvest volume control constraints, can also be incorporated in the analysis if needed to meet managerial objectives. The number of harvest control constraints are the same in either formulation. The preceding emphasis on the number of activities and constraints needed by each model to formulate a solution is because of the concern that the resulting formulations do not exceed computer limitations.

## Linear Programing

As mentioned earlier, each formulation for solving the optimum scheduling of stands in the forest can have two pricing options if the objective involves an economic criterion. The first, and historically the most widely used, option is founded on the principle that for a given stand class, treatment plan, and planning period, the unit price received is constant regardless of the volume cut. If this assumption is acceptable, the operations research technique of linear programing can be applied to solve both the Model I and the Model II formulations. The works of Loucks (1964), Navon (1971), and Ware and Clutter (1971) represent the application of linear programing to solving Model I formulations. In their review, Johnson and Scheurman (1977) demonstrated how the simulation techniques of the area-volume check method, SORAC (Chappelle 1966) and SIMAC (Sassaman and others 1972) could all be reformulated into different forms of Model II. They also showed how Nautiyal and Pearse's (1967) formulation for solving the optimal even-aged conversion problem could also be formulated as another form of Model II.

## Mathematical Programing Techniques for Quadratic Objective Functions

The second pricing option assumes that for a given stand class, treatment plan, and planning period, the unit price received for an acre cut is a function of the volume cut, and the form is one that declines linearly as the acreage cut increases. If this assumption is acceptable and reasonable, then, theoretically, the operations research technique quadratic programing can be applied to solve both basic formulations. A limited and specialized algorithm form has also been developed that incorporates the quadratic objective function. The first example of this algorithm form was Walker's (1976) ECHO (Economic Harvest Optimization Model), which Johnson and Scheurman (1977) reported to be a special formulation of Model II. A revised form of this algorithm is incorporated in TREES (Tedder and others 1980). Hrubec and Navon (1976) have approximated the ECHO solution using separable linear programing.

## Goal Programing

Both Model I and Model II pertain to management programs that are concerned with single objectives such as maximization of volume production or of present net worth. For programs that are concerned with multiple objectives, the historical approach has been to pick one of them for optimization and treat the rest as constraints. In the area of multiple objective optimization techniques, goal programing has received some attention in forest related problems. Kao and Brodie (1979) have looked into the use of goal programing for scheduling the treatment of all stands within the forest. They examined the problem of maximizing present net worth (objective 1), of maintaining an even-flow of harvest (objective 2), and of producing a regulated forest structure at the end of the planning horizon (objective 3). In another study, Field and others<sup>4</sup> have demonstrated the trade off of maximizing volume while maintaining nondeclining evenflow on a southern forest.

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<sup>4</sup>Field, Richard C., Peter E. Dress, and James C. Fortson  
and goal programing procedures for timber harvest scheduling.

Goal programming is an extension of linear programming. The objective of goal programming is to minimize the weighted difference between the targeted value for each objective and the feasible value of each objective. The weights of priorities assigned to each objective are specified by the analyst and can be changed between runs to evaluate their influence upon the solution. Using a simple example, Kao and Brodie (1979) demonstrated that goal programming could produce reasonable, compromise solutions to problems with infeasible linear programming solutions due to conflicting objectives.

## Integer Programming

Both linear and goal programming can result in the acreage of a stand class being assigned to a number of alternative treatment plans. If it is desired that a stand class be assigned only one treatment plan, the appropriate optimization tool is either integer or integer-goal programming. To date, the computational limitations of these tools have precluded their widespread application to practical forestry problems (Bare and Norman 1969). Recent developments in optimization technology (Glover and others 1978), however, may provide for wider use of integer programming in the future.

## Evaluation of Optimization Methods for Answering Forest-wide Questions

### CHARACTERIZATION OF THE FOREST

In the forest-wide optimization methods, forests are usually characterized by very simple structures based on gross stand class aggregates. Aggregating stands in this fashion could cause problems in accurately predicting treatment plan responses because stand development models are usually nonlinear in form.

The disadvantage of having to aggregate stands has risen out of the need to minimize problem size so that solutions are computationally feasible, that is, the fewer stand classes, the smaller the problem. There is, however, another feasibility problem which may conflict with the objective of reducing problem size by defining few stand classes. When constrained optimization is used, it is often found that a large number of alternate treatment plans must be defined for each stand class in order to find an operationally feasible solution.

As an alternative to this historical approach, it seems equally possible to define a large number of more homogenous stands (or if need be, stand classes) and a few alternate treatment plans for each. These alternate treatment plans could be developed by determining the optimal treatment plan for several different rotation lengths. The choice of rotation lengths could be made through an intelligent examination of existing forest structure problems and by determining where resulting future yields may be most lacking. This approach would produce more defensible yield predictions, provide a link between stand level and forest-wide optimization, and would tie treatments to the basic management unit, the stand. Testing this approach for feasibility seems justified.

Problems associated with this disadvantage may be further minimized (or even eliminated) by the recent development of a new method for solving linear programming problems (Kolata 1979). This new technique is reportedly capable of solving much larger problems in shorter time periods than the currently used simplex method.

### GROWTH AND YIELD INFORMATION

The yield information for all of the forest-wide optimization methods is tabular in form. Therefore, any source of yield information can be utilized. As with the forest-wide simulation methods, thinning yields can be credited to the total harvest when they occur instead of at the final regeneration harvest cut.

## COMPUTATIONAL BURDEN

Forest-wide optimization methods usually take longer computer times to operate than their simulation counterparts (Johnson and Tedder<sup>1</sup>). Recent advances (Kolata 1979), however, may minimize or eliminate this disadvantage.

## FLEXIBILITY OF THE SOLUTION SPECIFICATION

All forest-wide optimization methods can evaluate both physical and economic objectives. Some of the advantages of optimization methods include:

1. Alternative treatment plans for each stand class can be evaluated in one run and an optimal plan determined.
2. Cutting priorities do not need to be specified nor assumed.
3. Complex constraint specifications can be incorporated into the analysis.
4. Solutions can usually be generalized because first order optimization conditions are stated or implied in the analysis method.

Finding feasible solutions, however, is often more difficult with optimization methods than with simulation methods (Johnson and Tedder<sup>1</sup>).

## OTHER CONSIDERATIONS

In many of the optimization methods, fractionization of stands can occur by assigning more than one treatment plan to the stand. The use of integer programming is a possible future solution to this problem.

# SIMULTANEOUS, OPTIMAL SOLUTION OF THE STAND- AND FOREST-LEVEL QUESTIONS

If a perfectly regulated forest structure exists or if harvest-level constraints are not important, then the joint optimization of stand and forest-level questions can be computed easily by determining each stand's optimal treatment plan and aggregating across the whole forest. Unfortunately, these conditions seldom exist and the manager is left with the problem of trying to determine suboptimal stand treatment plans that will meet forest-wide objectives and constraints.

Work on this joint optimization problem has only recently received some attention. Williams (1976) and Nazareth (1973) independently demonstrated an approach for joint optimization using an extremely simple forest structure. They found that it is the size of the joint optimization problem and not its structure that leads to solution difficulties. Many of the linear-programming harvest scheduling studies discussed above provide for optimization over several stand-level treatments. The dynamic programming solutions discussed above indicate a large, finite number of stand-level treatments. A single, large linear programming formulation, incorporating all of these stand-level treatments as activities, would be unmanageable in size. The problem is solvable, however, by decomposition--a process by which large linear programming problems can be partitioned into smaller problems whose solutions can be aggregated to provide the solution to the original large problem.

For the joint stand-level and harvest scheduling problem, the process is described as follows:

1. Solve the dynamic programming stand-level treatment network to find the optimal stand-level treatment sequence and the optimal sequence of treatments for every other potential stand condition. A large number of potential stand treatment activities are thus defined.
2. Enter the optimal sequence of stand-level treatments as activities in a constrained linear programming harvest scheduling problem and solve for the optimum harvest schedule. This solution is usually not the optimum harvest schedule over all possible stand-level treatments as there are probably non-optimal stand-level treatments that enhance the constrained objective function. For example, heavier than optimal thinning or early final harvest can provide volume in periods where available volumes from the optimal stand solutions are constraining.
3. A decomposition is performed whereby new activities are selected using information from the first solution as a starting point. A new optimum harvest schedule is formed using these activities.
4. Successive decompositions are performed with the process terminating after a prespecified number of decompositions or when the objective function enhancement between decompositions is trivial.

Because this joint optimization technique combines both dynamic and linear programming, it also exhibits their strengths and weaknesses. This technique has been used to jointly determine the optimal thinning plans, rotation lengths, and the harvest schedule for a forest of simple structure and a single species (Nazareth 1973; Williams 1976). While theoretically feasible, incorporation of optimal fertilization plans, species mixes, and diameter distribution for more complex forest structures has not yet been demonstrated. The recent advance in linear programming technology (Kolata 1979) may, however, eliminate the need for decomposition to solve this or even more complex problems. In any case, the linear programming portion of the method still would cause the fractionalization of stands (classes).

## SUMMARY

The assignment of the best (or optimal) treatment plan to every stand in the forest during each planning period is the basic decision each forest manager must make. Historically, this basic decision was made by partitioning it into several separate decisions. For even-aged management, these subordinate decisions have included the optimal planting density, thinning plan (and resulting diameter distributions), rotation length, fertilization plan and species mix for each stand, and the optimal schedule of stand treatments for the entire forest.

A number of optimizing and nonoptimizing decisionmaking tools have been developed to handle singly or jointly the planting density, thinning plan, and rotation length decisions at the stand level. While improvements in making these particular decisions might be possible using nonlinear programming, the quality of decisions now possible from existing tools appears adequate at this time. Stand-level decisions concerning optimal diameter distributions, fertilization plans, and species mixes, however, have not received adequate attention. Growth modeling limitations appear to be the major factors holding back development in this area.

At the forest level, a number of tools have also been developed to help in making stand treatment scheduling decisions. Additional research on the usage of integer programming to schedule entire stands and on the development of multiple objective optimization techniques would provide even greater analytical flexibility to the decisionmaker.

Finally, a technique for the simultaneous consideration of both stand-level and forest-wide decisions has recently been developed and reported (Nazareth 1973; Williams 1976). Expansion of this technology to include optimal diameter distributions, fertilization plans and species mixes for multiple objectives should ultimately lead to the solution of the basic managerial decision.

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## GLOSSARY

- Allowable cut - The amount of timber that can be removed in a planning period if the optimal schedule of stand treatments is fully applied to the forest.
- Allocation problem - The task of distributing limited resources to alternative uses.
- Area constraints - Constraints imposed in mathematical programming formulations to limit area harvested. A set of real area constraints is necessary to limit area harvested from an age class to an amount less than or equal to the area in the age class.
- Area control - A classical harvest scheduling technique that achieves a regulated forest in one rotation by harvesting equal areas in each time period. A variant of the method achieves constant equilibrium harvests by harvesting areas of constant long-run productivity during the conversion period. Area control stabilizes the area harvested during the conversion period, but volume harvest will fluctuate, dependent on the initial age class distribution.
- AREA - An algorithm (Sassaman and Chappelle 1967) for computing allowable cuts using the area control method.
- Area-volume-check - A repetitive classical harvest scheduling technique that attempts to resolve conflicts between volume, area, and time period of harvesting over the forest. An initial volume estimate of harvest is applied to the forest area and age-class distribution and the resulting areas harvested. Length of time for cutting the whole forest is then checked and if found unsatisfactory, the initial estimate is adjusted upwards or downwards until volume, area, and cutting time are judged satisfactory.
- ARVOL (ARea VOLume) - An algorithm (Chappelle 1966) for computing allowable cuts during the conversion period using the area-volume method.
- Backward recursion - A dynamic programming solution strategy that starts with the oldest stand age (the rotation age of the stand) and solves the optimal treatment plan to the youngest stand age. The advantage of the backward recursion method is that, if the stand should deviate from the optimal path, the network values already computed will show which new path is optimal to follow to the specified ending age. The disadvantage is that a new run must be made for each potential rotation length.
- Binary search - A technique for meeting objectives and constraints of a specified problem by repeated solutions until a feasible maximum or minimum solution is found. Approach is usually specific to a particular, rather than general problem specification. Binary search involves an efficient technique of identifying upper and lower bounds for the solution of each iteration and using the midpoint for the next iteration trial until a tolerance between upper and lower bound is met.
- Control theoretic optimization - A group of mathematical programming problems consisting of optimizing a set of control variables such as levels of thinning, timing of these thinnings, and final harvest. Techniques used are usually nonlinear programming or dynamic programming.
- Conversion period - The length of time required to apply the conversion strategy. Harvest volume may fluctuate during the conversion period if inventory must be accumulated or depleted to reach the fully regulated condition.
- Conversion strategy - A treatment plan designed to convert a forest from its present structure to a targeted structure.

Decomposition (Dantzig-Wolfe decomposition) - A process by which large linear programming problems can be disaggregated to provide the solution to the original large problem.

Deterministic analysis - Analysis conducted under the assumption that variables are single valued (that is, not governed by a probability distribution).

Diameter distribution schedule - A schedule for the number of trees by diameter class for each planning period.

Dynamic programming (DP) - a technique for solving mathematical programming problems using discrete numerical computation rather than simultaneous solution of continuous functions. Used in forestry for simultaneous solution of thinning plan and rotation length, and for other allocation problems.

ECHO (*E*conomic *H*arvest *O*ptimization) - An algorithm (Walker 1976) that uses a binary search technique for maximizing the present net worth of harvest schedule under the assumption of a downward sloping demand curve. The solution approximates a quadratic programming solution with specialized restrictions.

Feasible solutions - Any solution that satisfies all objectives and constraints in a problem specification. The optimal solution is the maximum or minimum valued feasible solution.

Fertilization plan - A program of fertilization that incorporates both the scheduling and the intensity of fertilization.

Forward recursion - A dynamic programming solution strategy that starts with the youngest stand age and determines for each possible rotation length over the planning horizon the optimal treatment plan for each rotation length. The advantage of the forward recursion method is that optimal treatment plans for all rotation lengths are determined in one run. In addition, its method of handling the growth of stands makes it easier to adapt stand development models to a dynamic programming formulation.

Goal programming (GP) - A technique for solving mathematical programming problems with multiple objectives and goals by minimizing the deviations from the goals. Two uses in forestry have been to determine compromise solutions for both multiple uses or harvest scheduling objectives.

Harvest scheduling technique - A method for determining when individual stands or stand classes should be thinned or final harvested in order to meet the forest owner's objectives.

Integer programming (IP) - A technique for solving mathematical programming problems where the variables are constrained to take integer values. Used in forestry for harvest scheduling problems where complete stands must be assigned to a particular treatment plan use.

Linear programming (LP) - A technique for solving mathematical programming problems in which the objective and constraint variables are linear. Used in forestry for solving resource allocation and harvest scheduling problems.

LP activities - A set of treatment plans that can be applied to a specified management unit.

LP constraints - Specified relationships that guarantee that a problem solution meets both the conditions desired (or imposed) by the manager and conditions necessary to meet the problem specification. (See Area constraints, Nonnegativity constraints, and Volume constraints for specific examples.)

Management unit - A group of stands usually defined by their initial stand age.

Marginal analysis - Analysis based on the economic optimization principle that marginal revenue should equal marginal cost, where marginal revenue is defined as the contribution to revenue of an additional unit produced and marginal cost is defined as the contribution to cost of an additional unit of production. So long as additional production contributes more to revenue than cost, production should be expanded.

Mathematical programming - A body of optimization techniques for solving problems with functionally specified objectives and constraints. The maximum or minimum value of the objective is found by a defined process in a finite number of steps. (See Linear programming, Goal programming, Integer programming, Nonlinear programming, Quadratic programming, and Dynamic programming for specific examples.)

Maximum forest rent rotation - The rotation for which average annual revenue minus cost is greatest. As an economic criterion for determining rotation length, this method has been strongly criticized for ignoring the opportunity cost incurred by locking up capital in forest investment.

Maximum MAI (Mean Annual Increment) Rotation - The rotation for which volume divided by age is maximum. This rotation provides the greatest physical yield in the long-run.

Maximum present net worth rotation - The rotation for which discounted revenue minus discounted cost is greatest evaluated on a single rotation basis. When used as a criterion, land value is implicitly zero and rotation is longer than the maximum soil expectation rotation.

Maximum soil expectation rotation - The rotation for which discounted revenue minus discounted cost is greatest evaluated on the basis of an infinite series of rotations. This criterion values land at its maximum value in even-aged forest use for given species, prices, costs, and discount rate. The rotation is shorter than an evaluation based on a single rotation.

Model I - A mathematical programming formulation for harvest scheduling in which the identity of the initial management units (or stand classes) is maintained throughout the analysis. Through use of a Model I formulation, management planning decisions are easily tied to the ground. Model I formulations have many activities and few constraints when compared with the comparable Model II formulations.

Model II - A mathematical programming formulation for harvest scheduling in which acres in initial management units are reclassified and aggregated into new management units as they are harvested. Model II formulations have fewer activities and more constraints than the comparable Model I formulation.

Network - Many dynamic programming problems are characterized as two or more dimensional graphs (networks). Even-aged stands can be characterized as developing in a graphical space characterized by: volume and age, volume and basal area or volume, age, and basal area. The number of variables (state descriptor variables) define the dimensions of the network. Dynamic programming analysis is undertaken using discrete intervals within the network.

Nonlinear programing (NLP) - A body of techniques for solving mathematical programing problems in which the objective and constraint variables are nonlinear. Used in forestry to determine uneven-aged diameter distribution schedules, conversion schedules for uneven-aged stands, and thinning plan and rotation age for even-aged forests.

Nonnegativity constraints - Constraints imposed in mathematical programing formulations to limit variables to zero or positive values because negative volumes or acres harvested are infeasible and have no meaning in the usual problem specification.

Normal yield tables - Tables that present, by site index, and stand age, the yield of "fully stocked," undisturbed natural stands.

Optimal solution - A solution which best, or most favorably, meets the specified objectives and constraints of management.

Planning horizon - The total number of planning periods in an analysis.

Planning period - The shortest time period (usually in years) of concern in an analysis. One-, five-, and ten-year planning periods are common.

Principle of optimality - The fundamental principle on which dynamic programing is based. The principle states that once the optimal path through the dynamic programing network has been determined from the initial state to a specific state, the optimal path through the remaining stages of the network depends only on the portion of the network not yet analyzed. The principle of optimality allows a great reduction in the amount of necessary calculations.

Pure simulation - A technique for examining the consequences of applying a set of management decisions and/or rules to a stand or forest. Alternative management decisions and/or rules can be examined through repeated application of the simulation process.

Quadratic programing - A special case of nonlinear programing in which the objective function is a quadratic form.

Regulated forest (even-aged) - Strictly defined, a forest with a stand age class distribution such that the area in each age class from age one to rotation age is equal to the total area of the forest divided by the rotation age. If this forest is harvested on an annual basis at the rate of area/rotation age, the yield and age-class structure will remain constant over time. The term in practice is applied to forest age-class distributions that are approximately uniformly balanced so as to approximate the above equilibrium conditions.

Rotation length - The period of time elapsing between the time a stand is regenerated and the time it is final harvested. Applicable to the even-aged management system.

SIMAC (Simulated Intensively Managed Allowable Cut) - An algorithm (Sassaman and others 1972) for computing the allowable cut over the planning horizon using binary search techniques. Age related cutting priorities, fixed allowable cuts (for the first ten decades), and an upper and lower bound on the allowable cut can all be specified by the user.

Single-tree/distance-dependent model - A stand development model that characterizes the stand by a plot composed of individual tree measurements, including tree coordinates. Using the tree coordinates, an individual tree's competitive status within the plot is computed by examining the nearness and size of surrounding trees. Models of this type have been developed for even-aged and uneven-aged, pure and mixed species stands.

- Single-tree/distance-independent model - A stand development model that characterizes the stand by a representative sample of individual tree measurements. Because tree coordinates are not used, a tree's competitive status within the stand is characterized by comparing the tree's size to all other trees in the stand or sample. Models of this type have been developed for even- and uneven-aged, pure and mixed species stands.
- SORAC (Short Run Allowable Cut) - A binary search algorithm (Chappelle and Sassaman 1968) for computing the allowable cut using area control, area-volume check, or both. The area control option provides greater flexibility than algorithm AREA, while the area-volume check method has incorporated into it a dynamic look-ahead feature which was not in ARVOL. The dynamic look-ahead feature simulates the continual updating process normally associated with timber management planning.
- Stand class - An aggregation of stands of similar species mix, site quality, and age into a common class.
- Stand development model - A mathematical model (often coded for computer use) that predicts the development of the tree component of a stand over time. Ideally, a stand development model should incorporate components for: gross growth, non-catastrophic mortality, regeneration, catastrophic losses, the introduction of managerial decisions, and the portrayal of the prediction result. (See Whole-stand/diameter-free, Whole-stand/diameter-function, Whole-stand/diameter-class, Single-tree/distance-dependent, and Single-tree/distance-independent models for definitions of the various types of stand development models.)
- State descriptors - A set of variables that uniquely characterize a condition (state) that a system may be in. It is used in dynamic programming analysis where even-aged stands are characterized by two-descriptors (age and volume or age and basal area) or three descriptors (age, basal area, and number of trees).
- Stochastic analysis - Analysis conducted under the assumption that variables can take one of several values with known probability (that is, governed by a probability distribution).
- TEVAP2 (Timber Evaluation And Planning 2) - A pure simulation algorithm (Myers 1974) for examining alternative allowable cut strategies over a single planning period. The algorithm projects individual stands under prespecified treatment plans as a basis for allowable cut determination.
- Thinning plan - A program of thinning that incorporates both the scheduling and the intensity of thinning.
- Treatment plan - A program of treatment(s) that incorporates both the scheduling and the intensity of application.
- TREES (Timber Resources Economic Estimation System) - A package of binary search techniques (Tedder and others 1980) which incorporates the alternatives of area control, volume control, SIMAC, SORAC, or ECHO type approaches. When compared to these other algorithms, TREES has been designed to provide a higher degree of flexibility for solving harvest scheduling problems.
- Volume constraints - Constraints imposed in mathematical programming formulations to limit volume harvested in each period.

- Volume control - A classical formula based harvest scheduling technique that uses forest volume growth, inventory or age class data in various combinations to determine conversion period allowable harvest. Volume harvested under this method will be constant for the planning period, but area harvested will vary depending on the initial age-class distribution. Formula results are gross approximations which were usually adjusted using area-volume-check techniques.
- Whole-stand/diameter-class model - A stand development model that provides average stand information by characterizing the stand as the number of trees in defined diameter classes (that is, dividing the stand diameter distribution into classes and predicting the number of trees in each class). Models of this type can be developed for even- or uneven-aged, natural or managed stands of pure or mixed species. They are initialized by site index, number of trees by diameter classes, and, perhaps, by additional stand variables such as stand age.
- Whole-stand/diameter-free model - A stand development model that provides average stand information, but does not provide diameter distribution data. Models of this type are usually for pure, managed or natural even-aged stands, and are initialized by site index and stand values such as stand age, average or quadratic diameter, and the basal area or total number of trees in the stand.
- Whole-stand/diameter-function model - A stand development model that provides average stand information through the use of smooth functions, such as the Weibull or beta distribution functions, to characterize the diameter distribution. Models of this type have usually been limited to pure unmanaged even-aged stands and are initialized by site index and stand age.





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Major stand-level and forest-wide decisions facing forest managers interested in applying even-aged management are identified. Traditional, current, or potentially useful techniques for making these decisions are reviewed. Emphasis is placed upon the way each method characterizes the stand or forest, the growth and yield information requirements of each method, the computational burden associated with each method, and the flexibility of each method and its adaptability to the form and complexity of the solution.

**KEYWORDS:** even-aged, forest management, management decisions, stand modeling, mathematical programing, optimization, simulation, harvest scheduling.

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